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Imposing Equality and Inequality Constraints on a Production Technology Using Bayesian Approach[1]

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ABSTRACT

Empirical studies in economics are often faced with the problem of violating restrictions dictated by economic theory. Obtaining estimates that either do not lend easily to economic interpretations, or that are not plausible, is common to investigations in the past. With inferences involving a production technology for example, equality and inequality restrictions are generally dictated by economic theory on the parameters of either the cost or the profit function. Generating a feasible solution has been found to be possible not with the sampling theory or classical, but with the Bayesian approach.

This paper demonstrates how Bayesian methodology can be used to accommodate equality and inequality restrictions on a cost-minimizing problem. Using the translog functional form, the equality restrictions involve conditions of symmetry and homogeneity in input prices, while the inequality constraints are concerned with monotonicity and concavity. The equality restrictions were readily handled, but the inequalities posed more of a problem. Issues like complicated integration and sampling from a truncated distribution have been found to deter analytical techniques; but with numerical methods, the study shows how obtaining posterior pdfs and deriving posterior means and standard deviations become easy. To impose equality restrictions, the Gibbs sampler was used. For the inequality constraints, the Metropolis-Hastings algorithm was found to be practical. Non-informative priors were used for all estimation. Estimates derived from both the Gibbs sampler and the Metropolis algorithm were found to be quite close, except for the corresponding standard deviations.

KEY WORDS AND PHRASES: cost functions, Markov Chain Monte Carlo, inequality constraints

1. INTRODUCTION

Estimation of a cost or a profit function using duality generally involves estimation of flexible functional forms such as the translog, normalized quadratic, generalized Leontief, and other forms. Unfortunately, these estimated functions frequently violate the regularity conditions (monotonicity, concavity, or convexity) implied by economic theory. One approach to this problem involves the imposition of parametric restrictions globally. However, the global imposition of regularity conditions forces many flexible functional forms to exhibit properties contrary to economic theory. For example, imposing global concavity on a generalised Leontief cost function will rule out complementarity between inputs (Diewert and Wales, 1987).

This paper makes use of the Bayesian approach, (such as those of Chalfant and Wallace (1992) and Terrell (1996)), which allows us to draw finite sample inferences concerning nonlinear functions of parameters. Empirical implementation of the Bayesian approach

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involves the use of Markov chain Monte Carlo (MCMC) simulation methods. Two of the algorithms used are the Gibbs sampler (when inequality constraints are not imposed) and the Metropolis-Hastings algorithm (when inequality constraints are included). Both can be used to draw samples from a marginal probability density indirectly, without having to derive the density itself. This methodology is important where posterior marginal densities can be difficult or impossible to derive analytically.

A system of cost and factor-share equations for the Australian merino woolgrowing industry is used to demonstrate how estimation, incorporating both equality and inequality restrictions can be done numerically via MCMC methods. Aside from the purpose of estimation, the MCMC algorithms demonstrates alternatives for carrying out Bayesian inference in seemingly unrelated regression equations with equality and inequality constraints on the coefficients.

The paper is presented with the following outline: Section 2 presents the empirical model taking the form of a seemingly unrelated regression (SUR) model, and which translate a standard economic model of producer behavior into a system of empirical cost and factor share equations. In Section 3 we describe iterative procedures for obtaining maximum likelihood estimates of the SUR model parameters. We also describe the Gibbs sampler and Metropolis-Hastings algorithm, and the manner in which monotonicity and curvature restrictions can be imposed. In Section 4 the data and the variables used are discussed. Estimation results like parameter estimates, predicted factor shares, and estimated input-price elasticities are presented in Section 5. Input-price elasticities are useful for feeding into studies which examine the welfare implications of policy decisions and technical change. The paper concludes with Section 6.

2. MODEL

Our model assumes that the technological possibilities faced by the firm can be expressed by the cost function:

$$(1) \qquad C(\mathbf{w}, q) \equiv \min_{\mathbf{x}} \{\mathbf{w}'\mathbf{x} : f(\mathbf{x}) \geq q, \mathbf{x} \geq \mathbf{0}\}$$

where $\mathbf{x}$ is an $I \times 1$ vector of inputs, $\mathbf{w}$ is an $I \times 1$ vector of input prices and q is scalar output. The cost function is assumed nonnegative for all positive prices and output, and linearly homogenous, nondecreasing (i.e., monotonic), concave and continuous in prices (Chambers, 1988). Also, the Hessian matrix of second-order price derivatives is symmetric. This paper is concerned with monotonicity and concavity, and the method in which these properties can be imposed locally on an estimated flexible functional form.

This study made use of the translog functional form of Christensen, Jorgensen and Lau (1971), assuming a constant returns to scale. Following the work of O'Donnell and Woodland (1991) we express the cost function as

$$(2) \qquad \ln(C/q) = \alpha_0 + \alpha_T T + \sum_{i=1}^{I} \alpha_i \ln(w_i) + .5 \sum_{i=1}^{I} \sum_{j=1}^{I} \alpha_{ij} \ln(w_i) \ln(w_j)$$

where C represents total costs, w_i represents the price of input i, and T is a time trend which is used to capture the effects of exogenous technical change. The factor share equations are obtained using Shephard's lemma:

$$(3) \qquad s_i = \alpha_i + \sum_{j=1}^{I} \alpha_{ij} \ln(w_j) \quad i = 1, \ldots, I$$

where s_i represents the cost share of input i. From equations (2) and (3) we see that our assumed form of technical change is Hicks neutral (i.e., factor shares are unaffected by technical change while unit costs decrease at a constant percentage rate). Some of the theoretical properties of the cost function (1) can be expressed in terms of the parameters appearing in equation (2). Specifically, linear homogeneity and symmetry will be satisfied if

$$(4) \qquad \sum_{i=1}^{I} \alpha_i = 1, \qquad \sum_{j=1}^{I} \alpha_{ij} = 0 \quad (i = 1, \ldots, I),$$

$$\text{and} \qquad \alpha_{ij} = \alpha_{ji} \quad (i, j = 1, \ldots, I).$$

Monotonicity will be satisfied if the estimated factor shares are positive, while concavity will be satisfied if the Hessian matrix of second-order derivatives is negative semi-definite (i.e., if and only if its eigenvalues are non-positive). With Bayesian estimation, the parameter space where monotonicity and concavity hold is denoted by Γ_2 while the unrestricted parameter space is denoted by Γ_1.

Our empirical model is obtained by embedding equations (2) and (3) in a stochastic framework. Incorporating stochastic terms and introducing the firm and time subscripts n and t ($n = 1, \ldots, N$ and $t = 1, \ldots, T$), our empirical model becomes

$$(5) \qquad \begin{aligned} s_{int} &= \alpha_i + \sum_{j=1}^{I} \alpha_{ij} \ln(w_{jnt}) + \varepsilon_{int} \qquad i = 1, \ldots, I\text{-}1 \\ \ln(C_{nt}/q_{nt}) &= \alpha_0 + \alpha_T T_{nt} + \sum_{i=1}^{I} \alpha_i \ln(w_{int}) + .5 \sum_{i=1}^{I} \sum_{j=1}^{I} \alpha_{ij} \ln(w_{int}) \ln(w_{jnt}) + \varepsilon_{Int} \end{aligned}$$

where ε_{int} $(i = 1, \ldots, I)$ represents statistical noise. One share equation is dropped to avoid singularity of the error covariance matrix. The share and cost equation errors are assumed to be independently and identically distributed over firms and time with

$$(6) \qquad E\{\varepsilon_{int}\} = 0$$

and

$$(7) \qquad \{\varepsilon_{int}\varepsilon_{mks}\} = \begin{cases} \sigma_{im} & \text{if } n = k \text{ and } t = s \\ 0 & \text{otherwise.} \end{cases}$$

The stochastic assumptions allow for within-firm contemporaneous correlation between the disturbances ε_{int}. The cost function combines errors that vary over time and firms with any time-specific uncertainty that may exist. Hence the cost function does not have a complicated error components structure.

3. ESTIMATION

Four methods for estimating the parameters of the model given by equations (4) to (7) are discussed: two equivalent methods for obtaining maximum likelihood estimates, and two MCMC algorithms (the Gibbs sampler and the Metropolis-Hastings algorithm). The maximum likelihood methods do not allow for the imposition of monotonicity or concavity constraints. The Gibbs sampler also does not impose these properties, but the estimates are used as benchmark by which to judge the maximum likelihood and the Metropolis-Hastings algorithm estimates. The Metropolis-Hastings algorithm demonstrates how MCMC methods can be used to ensure that monotonicity and concavity are satisfied.

3.1 Maximum Likelihood Estimation

The system of equations given by (5) can be written:

(8) $y_{int} = \mathbf{x}_{int}'\beta_i + \varepsilon_{int}$ $\quad i = 1, ..., 4$

where $y_{int} = s_{int}$ $\quad i = 1, ..., 3$

$y_{4nt} = \ln(C_{nt}/q_{nt})$

(9) $\beta_i = (\alpha_i, \alpha_{i1}, ..., \alpha_{i4})'$ $\quad i = 1, ..., 3$

(10) $\beta_4 = (\alpha_0, \alpha_T, \alpha_1, ..., \alpha_4, \alpha_{11}, \alpha_{12}, ..., \alpha_{14}, \alpha_{22}, \alpha_{23}, ..., \alpha_{44})'$

and the definitions of the $\mathbf{x}_{int}$ conform to the definitions of the β_i. Notice from equations (9) and (10) that the β_i vectors have many elements in common. The restrictions given by equation (4) and the restrictions implicit in equations (9) and (10) together mean that only 11 of the 31 parameters in the β_i vectors are 'free'. Those that are redundant can be obtained from the other parameters and the restrictions.

If we stack equation (8) by firm, time period and then by equation, we have

$$(11) \quad \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{bmatrix} = \begin{bmatrix} X_1 & & & \\ & X_2 & & \\ & & X_3 & \\ & & & X_4 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

where $\mathbf{y}_i = (y_{i11}, y_{i21}, ..., y_{iN1}, y_{i12}, y_{i22}, ..., y_{iN2}, \dots, y_{iNT})'$ is $NT \times 1$ for all i, and X_i and ε_i are similarly defined, although it is worth noting that X_i is $NT \times 5$ for $i = 1, ..., 3$ and X_4 is $NT \times 16$. More compactly, we can write

(12) $\mathbf{y} = X\beta + \varepsilon$

The parametric restrictions implied by equations (4), (9) and (10) and our assumptions concerning the error vector ε can also be written more compactly as:

(13) $R\beta = \mathbf{r}$

(14) $E\{\varepsilon\} = \mathbf{0}$

and

(15) $E\{\varepsilon\varepsilon'\} = \Omega = \Sigma \otimes I_{NT}$

where $\Sigma = [\sigma_{im}]$ and R and $\mathbf{r}$ are known matrices of order (20×31) and (20×1) respectively. The model given by equations (12) to (15) is a standard restricted SUR model (see Judge *et al.*,1985, pp.469-473).

To obtain maximum likelihood estimates we note that the restricted Generalized Least Squares (GLS) estimator for β is

$$(16) \quad \tilde{\beta} = \hat{\beta} + CR'(RCR')^{-1}(\mathbf{r} - R\hat{\beta})$$

where $C = [X'(\Sigma^{-1} \otimes I_{NT})X]^{-1}$ and $\hat{\mathbf{b}} = CX'(\Sigma^{-1} \otimes I_{NT})\mathbf{y}$ is the unrestricted GLS estimator. In practice, restricted Estimated Generalized Least Squares (EGLS) estimates can be obtained by replacing Σ in equation (16) with an estimator, constructed using restricted or unrestricted OLS residuals. Another estimate of Σ can then be obtained by replacing Σ with a new estimator based on the restricted EGLS residuals (rather than OLS residuals). We can continue to update our estimates of Σ in an iterative way and, if the disturbances are multivariate normal, this iterative process will yield maximum likelihood estimates.

The iterative process just presented can be time-consuming if the number of restrictions to be imposed and parameters to be estimated at each step is large. An alternative but equivalent estimation procedure involves maximum likelihood estimation of the subset of 11 free parameters in β. After convergence, the remaining 20 maximum likelihood estimates are derived using the 20 parametric restrictions $R\beta = \mathbf{r}$. To implement the procedure we rearrange the rows of β and the columns of X and R in such a way that equations (12) and (13) can be written in the following partitioned form:

$$(17) \quad \mathbf{y} = X\beta + \varepsilon = [X_1 \ \ X_2]\begin{bmatrix} \eta \\ \gamma \end{bmatrix} + \varepsilon$$

$$(18) \quad R\beta = [R_1 \ \ R_2]\begin{bmatrix} \eta \\ \gamma \end{bmatrix} = \mathbf{r}$$

where X_1, X_2, R_1, R_2, γ and η are $NT \times 20$, $NT \times 11$, 20×20, 20×11, 11×1 and 20×1 respectively. The vector γ contains the subset of 11 free parameters to be estimated in the first stage, and η contains the 20 remaining parameters in β which will be estimated using estimates of γ and the following equivalent form of equation (18):

$$(19) \quad \eta = R_1^{-1}(\mathbf{r} - R_2\gamma).$$

The vector γ of free parameters contains parameters which cannot be obtained from other parameters and the restrictions. To estimate γ we use (19) to rewrite (17) in the form:

$$(20) \quad \mathbf{y}^* = X^*\gamma + \varepsilon$$

where $\mathbf{y}^* = \mathbf{y} - X_1R_1^{-1}\mathbf{r}$ and $X^* = X_2 - X_1R_1^{-1}R_2$. The model given by (20), (14) and (15) is an unrestricted SUR model, with (unrestricted) GLS estimator for γ given by

$$(21) \quad \hat{\gamma} = C^*X^{*\prime}(\Sigma^{-1} \otimes I_{NT})\mathbf{y}^*$$

where $C^* = [X^{*\prime}(\Sigma^{-1} \otimes I_{NT})X^*]^{-1}$. In practice, EGLS estimates can be obtained by replacing Σ with an estimator constructed using OLS residuals and if the disturbances are multivariate normal, a maximum likelihood estimate for γ can be obtained using the iterative procedure described above.

3.2 Bayesian Estimation

We begin by stating Bayes Theorem:

$$(22) \quad f(\gamma, \Sigma \mid y^*) \propto L(y^* \mid \gamma, \Sigma)\, p(\gamma, \Sigma)$$

where $\propto$ denotes 'proportional to', $f(\gamma, \Sigma \mid y^*)$ is the posterior joint density function for γ and Σ given y^* (the posterior density summarises all the information about γ and Σ after the sample y^* has been observed), $L(y^* \mid \gamma, \Sigma)$ is the likelihood function (summarising all the sample information), and $p(\gamma, \Sigma)$ is the prior density function for γ and Σ (summarising the nonsample information about γ and Σ). Our interest lies in the posterior density $f(\gamma, \Sigma \mid y^*)$ and characteristics (eg. means and variances) of marginal densities which can be derived from it.

The Bayesian treatment of the unrestricted SUR model begins with the assumption that ε is multivariate normal. Under this assumption the likelihood function is (Judge *et al*, p.478)

$$(23) \quad L(y^* \mid \gamma, \Sigma) \propto |\Sigma|^{-NT/2} \exp[-.5\, tr(A\Sigma^{-1})]$$

where A is the 4×4 symmetric matrix with $(i,j)^{\text{th}}$ element $a_{ij} = (y_i^* - X_i^* \gamma)'(y_j^* - X_j^* \gamma)$, and y_i^* and X_i^* are sub-vectors and matrices of y and X. In addition, we use a noninformative joint prior:

$$(24) \quad p(\gamma, \Sigma) = p(\gamma)\, p(\Sigma)\, I(\gamma \in \Gamma_s) \qquad s = 1, 2$$

where $p(\gamma) \propto$ constant, $p(\Sigma) \propto |\Sigma|^{-(I+1)/2}$ is the limiting form of a Wishart density, the Γ_s are the sets of permissible parameter values when monotonicity and concavity information is (s = 2) and is not (s = 1) available. We choose a noninformative prior because it allows us to better compare our maximum likelihood results with our Bayesian results, whether or not monotonicity and concavity information is available. The same is true of the joint posterior density (Judge *et al*, p.479) written as:

$$(25) \quad f(\gamma, \Sigma \mid y^*) \propto |\Sigma|^{-(NT+I+1)/2} \exp[-.5(y^* - X^*\gamma)'(\Sigma^{-1} \otimes I_{NT})(y^* - X^*\gamma)]\, I(\gamma \in \Gamma_s) \quad s = 1, 2$$

$$(26) \quad \propto |\Sigma|^{-(NT+I+1)/2} \exp[-.5\, \text{tr}(A\Sigma^{-1})]\, I(\gamma \in \Gamma_s) \quad s = 1, 2.$$

Of interest are the posterior marginal densities of the elements of γ, and the means and standard deviations of these posterior densities. Unfortunately, these results cannot be obtained from equations (25) and (26) analytically. Instead, we must use MCMC methods to draw a sample from the posterior joint density $f(\gamma \mid y^*)$. We then use these sample observations to estimate the moments of the marginal densities of the elements of γ. The two MCMC algorithms we use to generate these samples are the Gibbs sampler and Metropolis-Hastings algorithm.

The Gibbs Sampler

The Gibbs sampler was used for Bayesian estimation without monotonicity and concavity imposed. That is, the parameter space for γ was the unrestricted space Γ_1. The Gibbs sampler is an algorithm which effectively samples from $f(\gamma \mid y^*)$ by iterating as follows:

Step 1: Specify starting values γ^0, Σ^0. Set $i = 0$.
Step 2: Generate γ^{i+1} from $f(\gamma \mid \Sigma^i, y^*)$
Step 3: Generate Σ^{i+1} from $f(\Sigma \mid \gamma^{i+1}, y^*)$
Step 4: Set $i = i+1$ and go to Step 2.

This iteration scheme produces a chain, $\gamma^1, \Sigma^1, \gamma^2, \Sigma^2, \ldots$, with the property that, for large k, γ^{k+1} is effectively a sample point from $f(\gamma \mid y^*)$ (in this case the chain is said to have 'converged'). Thus, in practice, $\gamma^{k+1}, \ldots, \gamma^{k+m}$ can be regarded as a sample from $f(\gamma \mid y^*)$.

We obtain the kernel of the conditional posterior pdf using equation (25) and viewing Σ as a constant, to get

$$(27) \quad f(\gamma \mid \Sigma, y^*) \propto \exp[-.5(\gamma - \hat{\gamma})' X^{*\prime}(\Sigma^{-1} \otimes I_{NT}) X^*(\gamma - \hat{\gamma})] \, I(\gamma \in \Gamma_1)$$

Terrell did a modification in imposing constraints over a specified grid of prices: for each parameter vector generated by the Gibbs sampler (ie. for each γ^k), monotonicity and concavity constraints are evaluated at each price point in the grid. The parameter vector is included in the sample if the constraints hold and rejected otherwise. This modification has the effect of changing the conditional density in equation (27) to a truncated multivariate normal density that is only positive in the region Γ_2. It is often necessary to generate extremely large numbers of parameter vectors before obtaining just one vector that can be included in the sample; hence the Metropolis-Hastings algorithm which does not suffer this disadvantage, was used to impose the inequality constraints due to monotonicity and concavity.

The Metropolis-Hastings Algorithm

A Metropolis-Hastings algorithm which allows us to impose monotonicity and concavity at a particular set of prices is outlined as follows:

Step 1: Specify an arbitrary starting value γ^0 which satisfies the constraints. Set $i = 0$.
Step 2: Given the current value γ^i, use a symmetric transition density $q(\gamma^i, \gamma^c)$ to generate a candidate for the next value in the sequence, γ^c.
Step 3: Use the candidate value γ^c to evaluate the monotonicity and concavity constraints at the specified prices. If any constraints are violated set $\alpha(\gamma^i, \gamma^c) = 0$ and go to Step 5.
Step 4: Calculate $\alpha(\gamma^i, \gamma^c) = \min(g(\gamma^c)/g(\gamma^i)), 1)$ where $g(\gamma)$ is the kernel of $f(\gamma \mid y^*)$.
Step 5: Generate an independent uniform random variable U from the interval [0, 1].
Step 6: Set $\gamma^{i+1} = \begin{cases} \gamma^c \text{ if } U < \alpha(\gamma^i, \gamma^c) \\ \gamma^i \text{ if } U \geq \alpha(\gamma^i, \gamma^c) \end{cases}$
Step 7: Set $i = i+1$ and go to Step 2.

Again, this iteration scheme produces a chain, $\gamma^1, \gamma^2, \ldots,$ with the property that, for large k, γ^{k+1} is effectively a sample point from $f(\gamma \mid y^*)$. Thus, the sequence $\gamma^{k+1}, \ldots, \gamma^{k+m}$ can once again be regarded as a sample from $f(\gamma \mid y^*)$. Further, this sequence satisfies monotonicity and concavity at the specified prices. Notice from Steps 1, 2 and 4 that in order to make the Metropolis-Hastings algorithm operational we need an arbitrary starting value γ^0 which satisfies the constraints. For starting values we used $\alpha_i = 0.25$ ($i = 1, \ldots, 4$) and $\alpha_{ij} = 0$ for all $i \neq j$. All other parameters were set equal to their maximum likelihood estimates. These starting values satisfy monotonicity and concavity but may be some distance from the mean of $f(\gamma \mid y^*)$, and hence a reasonably long burn-in period is needed to ensure the convergence of the MCMC chain). The transition density $q(\gamma^i, \gamma^c)$ is assumed to be multivariate normal with mean γ^i and covariance matrix $[X^{*\prime}(\hat{\Sigma}^{-1} \otimes I_{NT})X^*]^{-1}$ (the estimated covariance matrix of the restricted SUR estimator $\hat{\gamma}$). The kernel $g(\gamma)$ of the marginal density $f(\gamma \mid y^*)$ can be obtained by integrating Σ out of the joint posterior (26) (see Judge *et al*, p.479):

(28) $(\gamma \mid y^*) \propto |A|^{-NT/2} I(\gamma \in \Gamma_2) = g(\gamma)$.

4. DATA AND VARIABLES

The data were originally collected by the Australian Bureau of Agricultural Economics (ABARE) as part of its Australian Sheep Industry Surveys (ASIS). Our sample consists of 310 time-series and cross-section observations on Australian merino woolgrowers, covering the periods 1952-53 to 1962-63 and 1964-65 to 1975-76, gathered by the Australian Bureau of Agricultural Economics (ABARE). Each observation in the original data set is a record of the average financial and physical characteristics of a group of firms. These observations were used to construct observations on output (q), total cost (C), input prices (**w**) and input quantities. Inputs were grouped into one of four broad categories: land, capital, livestock and other inputs (including labour, equipment, materials and services).

5. RESULTS

The results were generated using the econometric package SHAZAM.

Parameter Estimates

Maximum likelihood estimates of the structural parameters β are presented in Table 1, along with the means of the Bayesian samples obtained with and without the inequality constraints imposed, using the Gibbs sampler and the Metropolis-Hastings algorithm respectively. Note that all coefficients are statistically different from zero at usual levels of significance. The strong similarity between the maximum likelihood and Gibbs estimates presented in Table 1 reflects our use of a noninformative prior. The standard deviations of the Gibbs samples are slightly higher than the estimated standard errors of the maximum likelihood estimates, and this is expected since unlike the standard deviations of the Gibbs samples, the estimated standard errors of the maximum likelihood estimates do not account for the uncertainty associated with the estimation of the variance-covariance matrix Σ. For this reason, and because the maximum likelihood and Gibbs estimates are very similar, the maximum likelihood estimates are not considered henceforth. Finally, there is a reasonable similarity between the Gibbs and Metropolis-Hastings estimates presented in Table 1.

Predicted Factor Shares

Monotonicity requires that the predicted cost shares be positive. The distributions of the predicted factor shares were uniformly found to lie between zero and one, indicating that monotonicity was satisfied without the imposition of constraints.

Elasticities

Table 2 reports the means and standard deviations of the estimated pdfs of input price elasticities calculated at the quantity-weighted average of all input prices in the sample. The means of all own-price elasticities are correctly signed indicating that all input demands are inelastic with respect to their own prices. Moreover, the only own-price elasticity affected by the imposition of the constraints is the own-price elasticity for livestock. Another observation is that the standard deviations of the constrained and unconstrained probability density functions are generally similar, except the standard deviation of the own-price elasticity for livestock. Finally, the two cross-price elasticities which measure the relationships between the prices and quantities of livestock and other inputs undergo a sign reversal with the imposition of the constraints. Thus, livestock and other inputs appear to be substitutes in production. It is also interesting to note that the imposition of concavity changed the coefficient estimates very little despite the fact that the unconstrained estimates led to concavity violations at several price vectors. Furthermore, small differences in the coefficient estimates have led to much greater differences in a few of the elasticities.

Table 1: Parameter Estimates

		ML[a]	Gibbs[b] (no inequality constraints)	Metropolis-Hastings[b] (inequality constraints imposed)
Constant		-0.595	-0.597	-0.840
		(0.058)	(0.062)	(0.050)
α_1	Land	0.250	0.250	0.251
		(0.005)	(0.006)	(0.006)
α_2	Capital	0.674	0.674	0.664
		(0.019)	(0.020)	(0.017)
α_3	Livestock	0.440	0.440	0.344
		(0.013)	(0.013)	(0.008)
α_{11}	Land/Land	0.023	0.023	0.023
		(0.001)	(0.001)	(0.001)
α_{12}	Land/Capital	0.018	0.018	0.018
		(0.001)	(0.001)	(0.001)
α_{13}	Land/Livestock	-0.006	-0.006	-0.006
		(0.001)	(0.001)	(0.001)
α_{22}	Capital/Capital	0.115	0.115	0.110
		(0.006)	(0.006)	(0.006)
α_{23}	Capital/Livestock	-0.007	-0.007	-0.006
		(0.002)	(0.003)	(0.002)
α_{33}	Livestock/Livestock	0.076	0.076	0.057
		(0.002)	(0.002)	(0.001)
α_T	Time	-0.032	-0.032	-0.033
		(0.002)	(0.003)	(0.002)

[a] Numbers in parentheses are estimated standard errors.

[b] Numbers in parentheses are standard deviations of the MCMC samples.

TABLE 2 Estimated Input-Price Elasticities Evaluated at Average Prices[a]

	Price of Land	Price of Capital	Price of Livestock	Price of Other Inputs
Gibbs (unconstrained)				
Qty of Land	-0.647 (0.011)	0.493 (0.010)	0.027 (0.007)	0.127 (0.018)
Qty of Capital	0.148 (0.003)	-0.314 (0.022)	0.072 (0.009)	0.094 (0.022)
Qty of Livestock	0.025 (0.007)	0.218 (0.027)	-0.126 (0.024)	-0.117 (0.034)
Qty of Other Inputs	0.022 (0.003)	0.053 (0.012)	-0.022 (0.006)	-0.053 (0.015)
Metropolis-Hastings (constrained)				
Qty of Land	-0.643 (0.011)	0.496 (0.011)	0.030 (0.008)	0.118 (0.018)
Qty of Capital	0.148 (0.003)	-0.333 (0.021)	0.077 (0.008)	0.108 (0.020)
Qty of Livestock	0.027 (0.007)	0.229 (0.023)	-0.326 (0.005)	0.070 (0.023)
Qty of Other Inputs	0.020 (0.003)	0.061 (0.011)	0.013 (0.004)	-0.094 (0.012)

[a] Numbers in parentheses are standard deviations of the MCMC samples.

6. CONCLUSION

This paper uses Bayesian methods to impose regularity conditions on a system of cost and factor share equations. The Bayesian methodology represents an alternative to conventional sampling theory techniques which can impose regularity, but typically destroy the flexibility properties of many of the more popular functional forms. For problems with a large number of inequality constraints, the Metropolis-Hastings algorithm is an MCMC technique with much greater practical usefulness. The study offers a number of opportunities for further research. The most interesting of these involves the specification of a more complex error structure. Other extensions include the use of alternative functional forms and relaxation of the assumption of constant returns to scale, and application of the methodology to the Philippine data.

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References

CASELLA, G. AND E.I. GEORGE. 1992, "Explaining the Gibbs Sampler"', *The American Statistician*, 46(3):167-173..

CHAMBERS, R.G. 1988, *Applied Production Analysis*, Cambridge University Press, Cambridge.

CHIB, S. AND E. GREENBERG. 1996, "Markov Chain Monte Carlo Methods in Econometrics", *Econometric Theory,* 12(1996):409-431.

CHRISTENSEN, L.R., D.W. JORGENSEN AND L.J. LAU. 1971, "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica*, 39, 255-256.

DIEWERT, W.E. AND T.J. WALES. 1987, "Flexible Functional Forms and Global Curvature Conditions", *Econometrica*, 55(1), 43-68.

GALLANT, A.R. AND E.H. GOLUB. 1984, "Imposing Curvature Restrictions on Flexible Functional Forms", *Journal of Econometrics*, 26, 295-321.

JUDGE, G.G., W.E. GRIFFITHS, R.C. HILL, H. LUTKEPOHL AND T.C. LEE. 1985, *The Theory and Practice of Econometrics* (2nd ed.), John Wiley, NY.

O'DONNELL, C.J. AND A.D. WOODLAND. 1995, "Estimation of Australian Wool and Lamb Production Technologies Under Uncertainty: An Error-Components Approach", *American Journal of Agricultural Economics*, 77(August), 552-565.

POPE, R.D. AND R.E. JUST. 1998, "Cost Function Estimation Under Risk Aversion", *American Journal of Agricultural Economics*, 80(May), 288-295.

TERRELL, D. 1996, "Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms", *Journal of Applied Econometrics*, 11(2):179-94.

ZELLNER, A. 1971, *An Introduction to Bayesian Inference in Econometrics*, John Wiley, NY.